1.1
Units

In order to make numerical records of our measurements of position, time, and mass we need to adopt a unit of length, a unit of time, and a unit of mass, so we can express our measurements as numerical multiples or fractions of these units. We will use the **metric system of units**, which is based on the meter as the unit of length, the second as the unit of time, and the kilogram as the unit of mass. These units of length, time, and mass, in conjunction with the unit of temperature and the unit of electric charge, are sufficient for the measurement of any physical quantity. Scientists and engineers refer to this set of units as the **International System of Units**, or **SI units**.

The unit of length is meter. A **meter** is defined as the length that light travels in vacuum in a time interval of 1/299,792,458 second.

The unit of time is second. One **second** is defined as the time required for 9,192,631,770 cycles of the microwave radiation during the transition of cesium atoms between two lowest energy states.

The standard of mass, the **kilogram**, is defined to be the mass of a particular cylinder of platinum-iridium alloy. The gram is 0.001 kilogram.

The larger and smaller units of a physical quantity can be introduced in terms of fundamental units. In the metric system these units are related to the fundamental units by multiples of 10 or $\frac{1}{10}$. Thus one kilometer (1 km) is 1000 meters, and one centimeter (1 cm) is $\frac{1}{100}$ meter. We usually express multiples of 10 or $\frac{1}{10}$ in exponential notation: $1000 = 10^3$, $\frac{1}{1000} = 10^{-3}$, and so on. With this notation, $1\ \text{km} = 10^3\ \text{m}$ and $1\ \text{cm} = 10^{-2}\ \text{m}$.

The names of the additional units are derived by adding a **prefix** to the name of the fundamental unit. For example, the prefix "kilo-," abbreviated k, means a unit larger by a factor of 1000; thus

$$1\ \text{km} = 10^3\text{m}$$
$$1\ \text{kg} = 10^3\text{g}$$
$$1\ \text{kiloWatt} = 1\ \text{kW} = 10^3\text{Watts} = 10^3\text{W}$$

Here are several examples of the use of multiples of 10 and their prefixes with the units of length, mass, and time.

Length

$$1\ \text{nanometer} = 1\ \text{nm} = 10^{-9}\ \text{m}$$
$$1\ \text{micrometer} = 1\ \mu\text{m} = 10^{-6}\ \text{m}$$
$$1\ \text{millimeter} = 1\ \text{mm} = 10^{-3}\ \text{m}$$
$$1\ \text{centimeter} = 1\ \text{cm} = 10^{-2}\ \text{m}$$
$$1\ \text{kilometer} = 1\ \text{km} = 10^{3}\ \text{m}$$

Mass

$$1\ \text{microgram} = 1\ \mu\text{g} = 10^{-6}\ \text{g} = 10^{-9}\ \text{kg}$$
$$1\ \text{milligram} = 1\ \text{mg} = 10^{-3}\ \text{gm} = 10^{-6}\ \text{kg}$$
$$1\ \text{gram} = 1\ \text{g} = 10^{-3}\ \text{kg}$$

Time

$$1\ \text{nanosecond} = 1\ \text{ns} = 10^{-9}\ \text{s}$$
$$1\ \text{microsecond} = 1\ \mu\text{s} = 10^{-6}\ \text{s}$$
$$1\ \text{millisecond} = 1\ \text{ms} = 10^{-3}\ \text{s}$$

TABLE 1.3 PREFIXES FOR UNITS

MULTIPLICATION FACTOR	PREFIX	SYMBOL
10^{21}	zetta	Z
10^{18}	exa	E
10^{15}	peta	P
10^{12}	tera	T
10^{9}	giga	G
10^{6}	mega	M
10^{3}	kilo	k
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p
10^{-15}	femto	f
10^{-18}	atto	a
10^{-21}	zepto	z

1.2
Unit Consistency and Conversions

Consistency of Units

In all the equations, the units on the left and the right sides of the equation must be consistent. The consistency is illustrated by the calculations in the example below.

Example 1: if a body moving with constant speed v travels a distance d in a time t, these quantities are related by the equation

$$d = vt$$

If *d* is measured in meters, then the product vt must also be in meters. We may write

$$10\ \mathrm{m} = \left(2\ \frac{\mathrm{m}}{s}\right)(5\ \mathrm{s})$$

Because the unit $1/s$ on the right side of the equation cancels the unit s, the product vt has units of meters, as it must.

In calculations, units are treated just like algebraic symbols with respect to multiplication and division. For example, to find the number of seconds in 3 min, we write

$$3\ \mathrm{min}\ = \ (3\ \mathrm{min})\left(\frac{60s}{1\mathrm{min}}\right) = 180\ s$$

It is a general rule that in any calculation with the equations, the units can be multiplied and divided as though they were algebraic quantities, and this automatically yields the correct units for the final result.

This requirement of consistency of units in the equations can be reformulated in a more general way as a requirement of consistency of dimensions. In this context, the **dimensions** of a physical quantity are said to be length, time, mass, or some product or ratio of these.

Associated with every measured or calculated quantity is a dimension. The units in which the quantities are expressed do not affect the dimension of the quantities: an area is still an area whether it is expressed in m^2 or ft^2.

Just as certain measurement standards are defined as fundamental quantities, a set of fundamental dimensions can be chosen based on independent measurement standards. For mechanical quantities, mass, length, and time are elementary and independent, so they can serve as fundamental dimensions. They are represented respectively by M, L, and T. We use square brackets [ ] to denote "the dimensionality of," so that $[x] = \mathrm{L}$ or $[t] = \mathrm{T}$.

The dimensions of area and volume are

$$[\mathrm{A}] = \mathrm{L}^2 \quad \text{and} \quad [\mathrm{V}] = L^3$$

The dimensions of speed (m/s) are

$$[v] = \frac{L}{T} = LT^{-1}$$

Any equation must be *dimensionally consistent*, that is, the dimensions of the two sides of the equation must be the same. For instance, we can test the consistency of the equation

$$d = vt$$

by examining the dimensions of the quantities appearing in this equation.

$$\begin{aligned} \mathrm{L} &= \mathrm{LT}^{-1}\mathrm{T} \\ &= \mathrm{L} \end{aligned}$$

Dimensions are often used in preliminary tests of the consistency of equations, where there is some suspicion of a mistake in the equation. A test of the consistency of dimensions tells us no more than a test of the consistency of units, but has the advantage that we need not commit ourselves to a particular choice of units. We bear in mind that if an equation fails this consistency test, it is proved wrong; but if it passes, it is not proved right.

Dimensions are sometimes used to find relationships between physical quantitees. Such a determination of the appropriate proportionality between powers of relevant quantiites is called **dimensional analysis**. Such analysis is performed by requiring the consistency of dimensions of units on each side of an equation.

In dimensional analysis, dimensions can be treated as algebraic quantities. For example, quantities can be added or subtracted only if they have the same dimensions. Furthermore, the terms on both sides of an equation must have the same dimensions.

For example, the distance *x* covered in a time *t* by an object starting from rest and moving with a constant acceleration *a* is known to be

$$x = \frac{1}{2}at^2$$

The dimensional consistency of this equation is verified as follows: Acceleration is measured in units m/s^2. Therefore, it has the dimensions

$$[a] = \frac{\mathrm{L}}{\mathrm{T}^2} = \mathrm{LT}^{-2},$$

and so, the equation in terms of dimensions is

$$\mathrm{L} = \mathrm{LT}^{-2}T^2 = L$$

DIMENSIONAL FORMULAE

S.No.	Quantity	Units MKS	Dimensional formula
1.	Area	Sq.m	$M^0L^2T^0$
2.	Volume	Cubic m	$M^0L^3T^0$
3.	Density	kg/m^3	$M^1L^{-3}T^0$
4.	Velocity	m/sec	$M^0L^1T^{-1}$
5.	Acceleration	m/sec^2	$M^0L^1T^{-2}$
6.	Force	newton	$M^1L^1T^{-2}$
7.	Momentum	kg.m/s	$M^1L^1T^{-1}$
8.	Impulse	kg.m/s	$M^1L^1T^{-1}$
9.	Pressure	Pascal	$M^1L^{-1}T^{-2}$
10.	Work, energy	joule	$M^1L^2T^{-2}$
11.	Power	watt	$M^1L^2T^{-3}$
12.	Moment of a force	newton×metre	$M^1L^2T^{-2}$
13.	Moment of a couple	newton×metre	$M^1L^2T^{-2}$
14.	Inertia	kg	$M^1L^0T^0$
15.	Moment of Inertia	$kg\times m^2$	$M^1L^2T^0$
16.	Angular velocity	Radian/s	$M^0L^0T^{-1}$
17.	Frequency	Hertz	$M^0L^0T^{-1}$
18.	Angular momentum	$kg\times m^2/s$ or Joule-Sec	$M^1L^2T^{-1}$
19.	Planck's constant	Joule×sec	$M^1L^2T^{-1}$
20.	Universal gravitation constant	$\frac{\text{newton}\times\text{m}^2}{\text{kg}^2}$	$M^{-1}L^3T^{-2}$
21.	Surface tension	newton/m	$M^1L^0T^{-2}$
22.	Coefficient of viscosity	$N\times s/m^2$	$M^1L^{-1}T^{-1}$
23.	Velocity gradient	Sec^{-1}	$M^0L^0T^{-1}$
24.	Spring constant	newton/m	$M^1L^0T^{-2}$
25.	Young's Modulus	newton/sq.m	$M^1L^{-1}T^{-2}$
26.	Universal gas constant	Joule/mole/°K	$M^0L^2T^{-2}K^{-1}$
27.	Specific heat	Joule /kg/°K	$M^0L^2T^{-2}K^{-1}$
28.	Coefficient of thermal Conductivity	Joule/s/m/°K	$M^1L^1T^{-3}K^{-1}$
29.	Latent heat	Joule/kg	$M^0L^2T^{-2}K^0$
30.	Coefficient of linear expansion	$(°C)^{-1}$	K^{-1}
31.	Charge	Coulomb	I^1T^1
32.	Current	Amp	I^1
33.	Potential difference	Volt	$M^1L^2I^{-1}T^{-3}$
34.	Resistance	Ohm	$M^1L^2I^{-2}T^{-3}$
35.	Resistivity	Ohm-meter	$M^1L^3I^{-2}T^{-3}$
36.	Capacitance	Farad	$M^{-1}L^{-2}I^2T^4$
37.	Inductance	Henry	$M^1L^2I^{-2}T^{-2}$
38.	Magnetic Induction	Tesla	$M^1I^{-1}T^{-2}$
39	Magnetic pole strength	Amp-m	I^1L^1
40.	Magnetic field strength	Amp/meter	I^1L^{-1}
41.	Magnetic flux	Weber	$M^1L^2I^{-1}T^{-2}$
42.	Magnetic moment	$amp\text{-}m^2$	I^1L^2
43.	Permeability	Henry/m	$M^1L^1I^{-2}T^{-2}$
44.	Permittivity	$Coul^2/n\text{-}m^2$	$M^{-1}L^{-3}I^2T^4$
45.	Electric field strength	Newton/coulomb (or) volt/m	$M^1LI^{-1}T^{-3}$